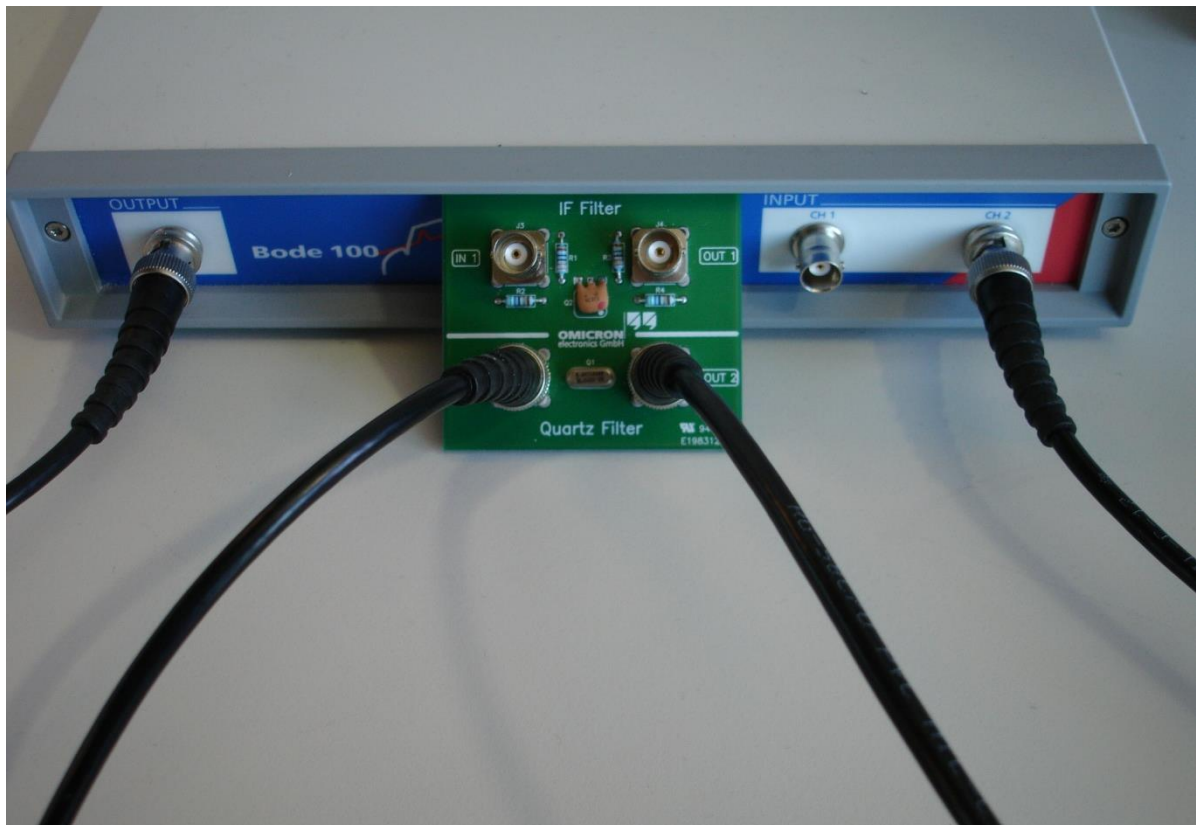


## Bode 100 - Application Note

# Equivalent Circuit Determination of Quartz Crystals



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**Note:** Basic procedures such as setting-up, adjusting and calibrating the Bode 100 are described in the Bode 100 user manual. You can download the Bode 100 user manual at [www.omicron-lab.com/bode-100/downloads#3](http://www.omicron-lab.com/bode-100/downloads#3)

**Note:** All measurements in this application note have been performed with the Bode Analyzer Suite V3.0. Use this version or a higher version to perform the measurements shown in this document. You can download the latest version at [www.omicron-lab.com/bode-100/downloads](http://www.omicron-lab.com/bode-100/downloads)

## 1 Executive Summary

This application note explains how to measure the equivalent circuit parameters of a quartz crystal with the Bode 100.

You will be given an insight on resonance frequency measurements and impedance measurements with the Bode 100.

## 2 Measurement Task

Each quartz crystal has resonance frequencies. This resonance behavior can be modeled with an equivalent electronic circuit. An equivalent description of a quartz crystal is given by the following circuit. It is valid in the frequency region of a single series - parallel resonance combination. These series – parallel resonance combinations occur at odd multiples of the fundamental series resonance frequency.

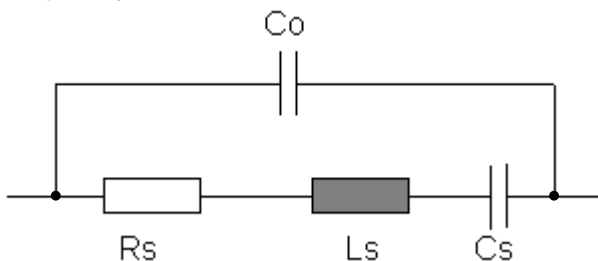


Figure 1: Quartz equivalent circuit model

Based on this model with the circuit parameters  $L_s$ ,  $C_s$ ,  $C_0$  and  $R_s$  the resonance frequencies and the quality factor  $Q$  can be calculated using the following equations:

$$\text{Series resonance frequency } f_s \quad f_s = \frac{1}{2\pi\sqrt{L_s C_s}} \quad (1)$$

$$\text{Parallel resonance frequency } f_p: \quad f_p = f_s \sqrt{1 + \frac{C_s}{C_0}} \approx f_s \left(1 + \frac{C_s}{2C_0}\right) \quad (2)$$

$$\text{Quality factor } Q \text{ at } f_s: \quad Q = \frac{1}{R_s} \sqrt{\frac{L_s}{C_s}} \quad (3)$$

With the Bode 100 the values for  $C_0$ ,  $f_s$  and  $f_p$  can be measured. When rewriting the equations from above  $C_s$  and  $L_s$  can be calculated from the results of these three measurements.

$$C_s = 2C_0 \left( \frac{f_p}{f_s} - 1 \right) \quad (4)$$

$$L_s = \frac{1}{4\pi^2 \cdot f_s^2 \cdot C_s} \quad (5)$$

The following pages show how to set up your equipment, how to use the measurement methods of the Bode 100 and formulas to calculate the equivalent circuit parameters of the delivered test object quartz crystal. Further on, the quality factor of the crystal will be measured.

## 3 Measurement Setup & Results

### 3.1 Measurement Equipment

- Bode 100 Vector Network Analyzer
- Test object (delivered test board with quartz filter)
- Measurement accessories (two BNC cables, BNC-thru adapter)

### 3.2 Measuring the Quartz

The used test object is the crystal mounted on the test PCB, which is delivered with Bode 100. In a first step we will determine the equivalent circuit values at the fundamental frequency.

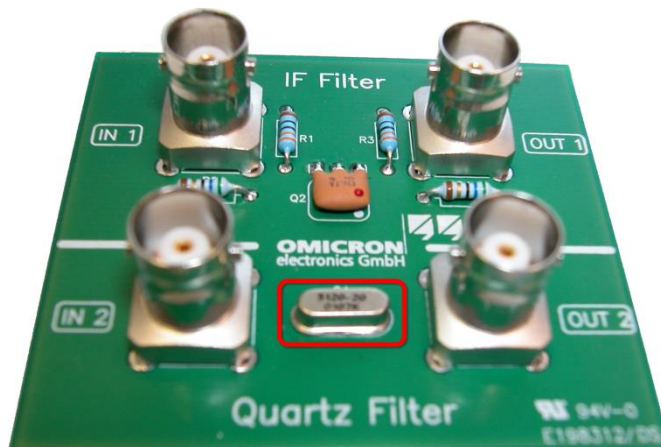


Figure 2: Test board with quartz filter

#### 3.2.1 Determination of the Parallel Capacitance

The quartz crystal has a nominal series resonance frequency of 12 MHz.

To measure  $C_0$  we need to measure the impedance of the crystal at a frequency which is well apart from the series- parallel resonant frequencies. For this 12 MHz crystal a measurement frequency of 10.5 MHz will be a good value. We expect a pure capacitive reactance as a measurement result.

The measurement can be performed with the Series-Thru measurement type as shown below.

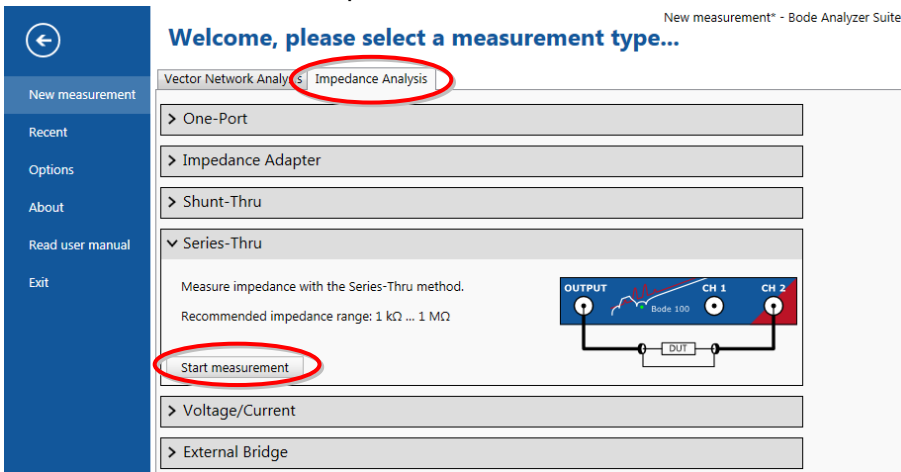


Figure 3: Start menu

The following settings are applied:

- Start frequency: 10 MHz
- Stop frequency: 15 MHz
- Receiver Bandwidth: 100 Hz
- Sweep mode: linear
- Output level: 0 dBm

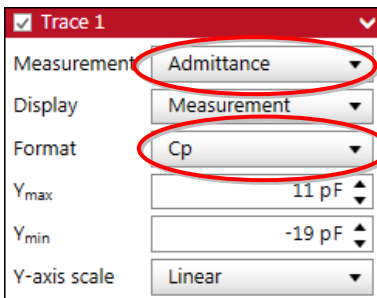


Figure 4: Settings Trace 1

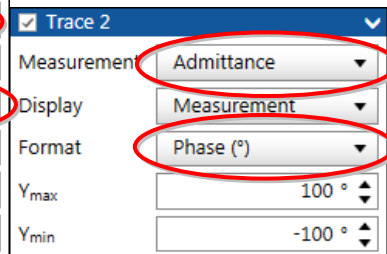


Figure 5: Settings Trace 2

In addition, a THRU calibration has to be performed to remove the cable influence. Details on the calibration can be found in the Bode 100 User Manual.

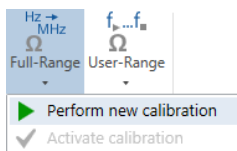


Figure 6: Full-Range Calibration icon



Figure 7: Cable setup while calibrating Thru

The input of the quartz filter is connected to the Bode 100 output and the output of the quartz filter to the channel 2 input of the Bode 100. (See Figure 8). Start the measurement and use right click "optimize" to display the measured data:



Figure 8: Setup with quartz filter

The following graph shows the result for the parallel capacitance  $C_0$ :

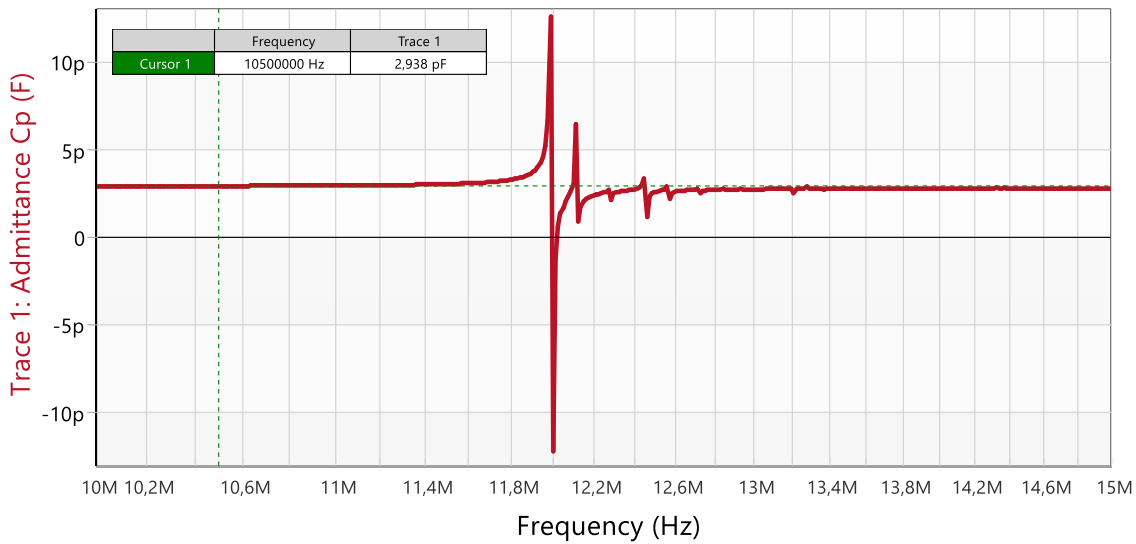


Figure 9: Measurement result Cp

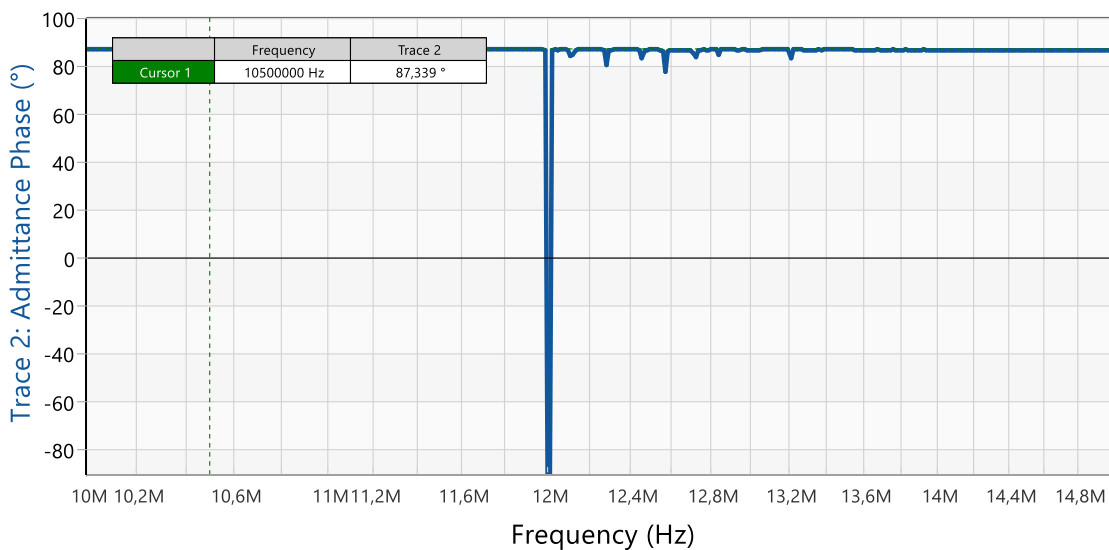


Figure 10: Measurement result Phase

Setting the cursor to the measurement frequency of 10.5 MHz leads to the following results:

	Frequency	Trace 1	Trace 2
<input checked="" type="checkbox"/> Cursor 1	10,5 MHz	2,938 pF	87,339 °

Figure 11: Cursor @ 10.5 MHz

The parallel capacitance beyond the resonance frequencies equals  $C_0 = 2.94$  pF. The value of the admittance phase with  $87.3^\circ$  shows nearly perfect capacitive behavior at the measurement frequency.

### 3.2.2 Measuring the Resonance Frequencies

To measure the series and parallel resonance frequency we keep the connection setup as it is.

Apply the following settings to Trace 1 in the Bode Analyzer Suite:

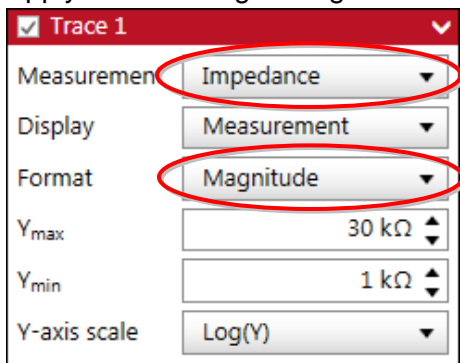


Figure 12: Settings Trace 1 - resonance frequencies

Now start a frequency sweep and optimize the view by right clicking on the chart and clicking "optimize".

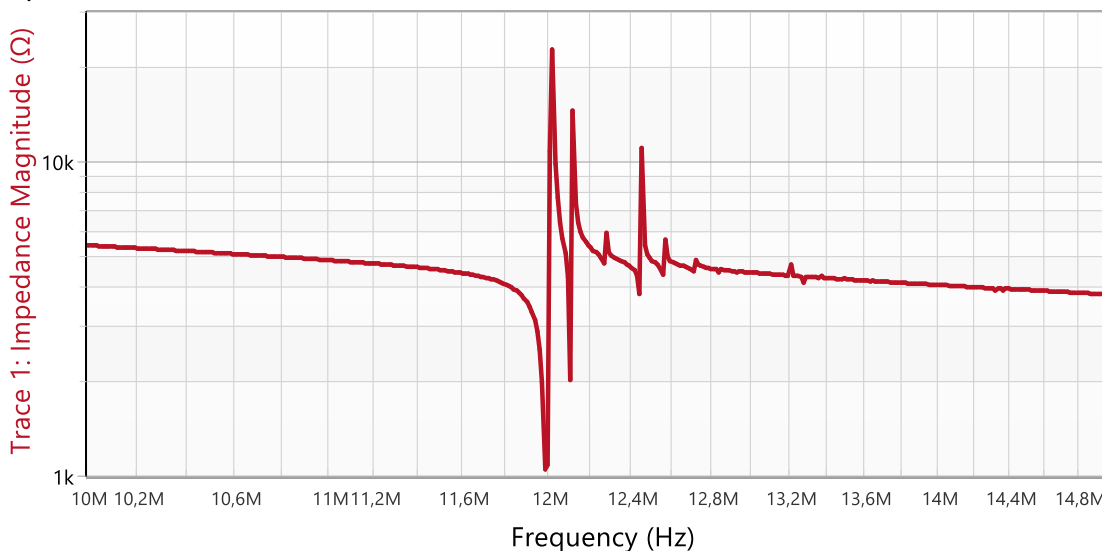


Figure 13: Measurement result - resonance frequencies

The transmission measurement shows multiple resonance frequencies. As we are only interested in the first parallel and series resonance we have to change the frequency range.

For that you can zoom into the graph. It is recommended to zoom into a window that has a frequency range from approx.  $f_{\min} = 11.97$  MHz to about  $f_{\max} = 12.04$  MHz for this measurement and device under test. To have more precise measuring points, we adjust the frequency by pressing the “Get from zoom” button on the left side.

**Note:** If you have trouble getting a precise and good looking curve, repeat optimizing, zooming and adjusting the frequency. Additionally you might want to raise the number of points.

Performing a measurement in this range shows the first series and parallel resonance frequencies:

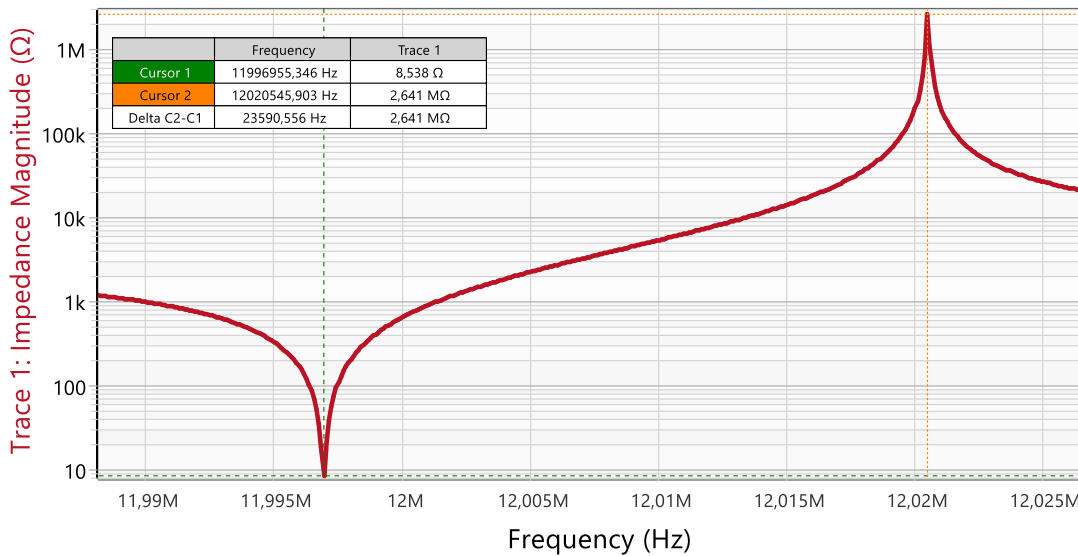


Figure 14: Measurement result - resonance frequencies – copy from zoom

Using two cursors and setting them to the maximum and minimum of the curve (use right click, cursor – jump to min/max) leads to the resonance frequencies:

	Frequency	Trace 1
<input checked="" type="checkbox"/> Cursor 1	11,996955 MHz	8,538 Ω
<input checked="" type="checkbox"/> Cursor 2	12,020546 MHz	2,641 MΩ
Delta C2-C1	23,591 kHz	2,641 MΩ

Figure 15: Cursor box

The measured resonance frequencies are:

$$f_s = 11.997 \text{ MHz}$$

$$f_p = 12.021 \text{ MHz}$$

Using the equations from 2 Measurement Task and the measured parallel & series capacitance as well as the series inductance can be calculated:

$$C_s = 2C_0 \left( \frac{f_p}{f_s} - 1 \right) = 11.72 \text{ fF} \quad (6)$$

$$L_s = \frac{1}{4\pi^2 \cdot f_s^2 \cdot C_s} = 15 \text{ mH} \quad (7)$$



With this measurement we additionally measured the last missing value ( $R_s$ ). If we look at Cursor 1 from above, we get an  $R_s$  of 8.54  $\Omega$ .

### 3.2.3 Determination of the Quality Factor

The quality factor  $Q$  of the quartz can be calculated using the measured values of the equivalent circuit model. The quality factor of the quartz is calculated as follows:

$$Q = \frac{1}{R_s} \cdot \sqrt{\frac{L_s}{C_s}} = \frac{1}{8.538\Omega} \cdot \sqrt{\frac{15\text{mH}}{11.72\text{fF}}} = 132587 \quad (8)$$

As a result of the performed measurements we are now able to describe the tested quartz crystal filter with a series – parallel equivalent circuit as follows:

$$\begin{aligned} C_0 &= 2.929 \text{ pF} \\ C_s &= 11.72 \text{ fF} \\ L_s &= 15 \text{ mH} \end{aligned}$$

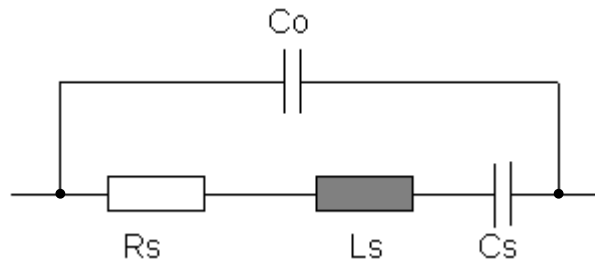


Figure 16: Quartz equivalent circuit model

## 4 Conclusion

The Bode 100 together with the Bode Analyzer Suite offers several useful functions to measure resonance frequencies or equivalent circuit parameters of any device under test.

The cursor functions help to find the exact measurement values at desired points. Predefined parameter measurements for series and parallel equivalent circuits reduce the users' calculation effort and enable easy and fast measurements.



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